DOCUMENT RESUME

ED 334 248 TM 016 812

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TITLE Canonical Correlation Analysis that Incorporates Measurement and Sampling Error Considerations.

PUB DATE 1 Apr 91
NOTE 34p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, April 3-7, 1991).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Correlation; Educational Research; Generalizability Theory; Mathematical Models; *Measurement Techniques; *Multivariate Analysis; Sampling
IDENTIFIERS *Canonical Redundancy Statistic; *Sampling Error

ABSTRACT

Multivariate methods are being used with increasing frequency in educational research because these methods control "experimentwise" error rate inflation, and because the methods best honor the nature of the reality to which the researcher wishes to generalize. This paper: explains the basic logic of canonical analysis; illustrates that canonical analysis is a general parametric analytic method subsuming other methods; and provides an example of one strategy that can be used to investigate the generalizability of multivariate results. Actual data from the Holzinger and Swineford (1939) study are used to contextualize the discussion. Five random samples of 301 cases were drawn from data modeled on the 10,000 cases of Holzinger and Swineford to explain the effects of sampling error. Eight tables and one figure present data, and a 49-item list of references is included. An appendix provides the Statistical Analysis System (SAS) program used. (SLD)

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Canonical Correlation Analysis that Incorporates Measurement and Sampling Error Considerations

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Paper presented at the annual meeting of the American Educational Research Association, Chicago, April 3, 1991.

ABSTRACT

Multivariate methods are being used with increasing frequency in educational research, because these methods control "experimentwise" error rate inflation, and because the methods best honor the nature of the reality to which the researcher wishes to generalize. This paper (a) explains the basic logic of canonical analysis; (b) illustrates that canonical analysis is a general parametric analytic method subsuming other methods; and (c) provides an example of one strategy that can be employed to investigate the generalizability of multivariate results. Actual data available from the widely known Holzinger and Swineford (1939) study are emplyed to make the discussion concrete.

Hinkle, Wiersma and Jurs (1979, p. 415) noted that "it is becoming increasingly important for behavioral scientists to understand multivariate procedures even if they do not use them in their own research." And recent empirical studies of research practice confirm that multivariate methods are employed with some regularity in behavioral research (Elmore & Woehlke, 1988).

There are two reasons why multivariate methods are so important, as noted by Fish (1988). First, multivariate methods limit the inflation of Type I "experimentwise" error rates. Most researchers are familiar with "testwise" alpha, which refers to the probability of making a Type I error on a *given hypothesis test*. "Experimentwise" error rate refers to the probability of having made a Type I error *anywhere within the study*. For example, if a researcher conducts a balanced three-way, factorial ANOVA, testing each of the three main effects, the three two-way interaction effects, and the single three-way interaction effect at the testwise .05 alpha level, the experimentwise error rate for the study will be:

$$\text{alpha}_{TW} = 1 - (1 - .05)^7 = 30.2\%.$$

The same difficulty can occur when multiple dependent variables are tested in a given study. The problem is that the researcher will know that an "experimentwise" error is likely, but will not know which of the statistically significant results are errors and which are not.

But an even more important reason to use multivariate methods is that multivariate methods best honor the reality to which the researcher is purportedly trying to generalize. Most researchers live in a reality "in which the researcher cares about multiple outcomes, in which most outcomes have multiple causes, and in which most causes have multiple effects" (Thompson, 1986, p. 9). We must use analytic models that honor our view of reality, or else we will arrive at interpretations that actually distort reality (Eason, 1991; Tatsuoka, 1973, p. 273).

Just as independent variables can interact to change results in ways that would go unnoticed if these interactions were not analyzed (Benton, 1991), so too dependent variables can interact with each other to create effects that would go unnoticed absent a multivariate analysis. Only multivariate analyses simultaneously consider the full network of variable relationships, and honor a reality in which all the variables can and often do simultaneously interact and influence each other. Thus, multivariate analyses can yield results that would remain undetected if univariate analyses (e.g., ANOVA, regression) were employed, as both Fish (1988) and Maxwell (in press) demonstrate using actual examples.

Canonical correlation analysis is a multivariate analytic method that subsumes other parametric methods (e.g., *t*-tests, ANOVA, ANCOVA, regression, discriminant analysis, MANOVA) as special cases (Knapp, 1978). Some researchers have found canonical analysis to be useful. For example, Wood and Erskine (1976) identified more than 30 published applications of these methods. More recently, Thompson (1989a) cited roughly 100 canonical applications reported during the last decade.

Although multivariate methods have enjoyed fairly widespread substantive usage (Thompson, 1989a; Wood & Erskine, 1976) since computers and statistical software became widely available, *multivariate methods also have been used in intriguing ways in measurement and assessment contexts*. For example, Merenda, Novack and Bonaventure (1976) reported a multivariate reliability analysis involving subtest scores from the California Test of Mental Maturity. Similarly, Sexton, McLean, Boyd, Thompson and McCormick (1988) reported results involving a multivariate concurrent validity analysis.

Unfortunately, as Nunnally (1978, p. 298) notes, "one tends to take advantage of chance in any situation where something is optimized from the data at hand." In fact, this capitalization occurs in all classical parametric methods, because all these methods (e.g., *t*-tests, ANOVA, regression, MANOVA) are least squares procedures that implicitly or explicitly (a) use weights, (b) focus on latent synthetic variables, and (c) yield effect sizes analogous to r^2, i.e., all classical analytic methods are correlational (Knapp, 1978; Thompson, 1988a).

But the problem of capitalizing on sampling error when multivariate methods are used is particularly acute, because the models being tested involve a larger system of parameter estimates. For example, the problem is particularly acute when factor analytic methods are employed, because "one has numerous possibilities for capitalizing on chance. Most extraction procedures, including principal factor solutions, reach their criterion by such capitalization. The same is true of rotational procedures, including those which rotate for simple structure" (Gorsuch, 1983, p. 330).

The purposes of the present article are (a) to explain the basic logic of canonical analysis in a concrete and accessible fashion; (b) to illustrate that canonical analysis is a general parametric analytic method subsuming other methods; and (c) to illustrate one method that can be employed to investigate the stability or the generalizability of results.

The Basic Logic of Canonical Calculations

Thompson (1984) notes that canonical correlation can be presented in bivariate terms. This conceptualization is appealing, because most researchers feel very comfortable thinking in terms of the familiar bivariate correlation coefficient. Table 1 presents a small data set that will be employed to illustrate the basic logic of canonical correlation analysis (CCA). Appendix A presents the SAS computer program used to analyze the data; readers may find it useful to replicate these analyses and to examine other results reported in the output but not presented here, given space limitations.

INSERT TABLE 1 ABOUT HERE.

The 12 cases of scores on each of two sets of scales ("CHA6" to "OTH2") were randomly sampled from a data base generated in one

of the "Heart Smart" studies, an offshoot of the Bogalusa Heart Study longitudinal examination of the origins of cardiovascular disease during childhood. The first set of scores involves actual values for these subjects on three scales, each with six items, measuring children's perceptions of the sources of their health: (a) Chance, i.e., random uncontrollable external factors ("CHA6"); (b) Internal, i.e., decisions or actions within one's own control ("INT6"); and (c) Powerful Others, i.e., external factors under the direct control of others, such as nurses or doctors ("OTH6"). The second set of scores ("CHA2" to "INT2") involved responses on six items (two per scale) from a different source, but purportedly measuring the same three constructs. The example is elaborated by Thompson, Webber and Berenson (1988), who present one of the several related analyses conducted with the full database.

Thus, the small heuristic Table 1 data set involves a concurrent validity context. A CCA invoked in an analytically similar measurement context but with a data set with a realistic sample size and different variables is presented by Sexton, McLean, Boyd, Thompson and McCormick (1988). Of course, CCA can be useful in addressing either substantive or measurement issues, but the latter context is perhaps more relevant to the focus of the journal.

Various analytic methods yield weights that are applied to variables to optimize some condition--such weights include beta weights, factor pattern coefficients, and discriminant function coefficients. These weights are all *equivalent* (e.g., Thompson & Borrello, 1985; Thompson, 1988), at least after a transformation in metric, but in canonical correlation analysis the weights are usually labelled standardized canonical function coefficients. It is difficult to fathom why the equivalent weights used in the various parametric methods are given different names, since the primary result is confusion and the illusion that parametric methods are different. The CCA function coefficients are applied to each individual's standardized data to yield the synthetic variables that are the basis for canonical analysis.

In regression only one set of weights is produced, but in canonical analysis several sets of weights and of the resulting synthetic variables can be created. These canonical functions are related to principal components, are uncorrelated or orthogonal, and can be rotated in various ways (Thompson, 1984; Thorndike, 1976). The number of functions that can be computed in a canonical analysis equals the number of variables in the smaller of the two variable sets. In the present example, since both sets of variables consisted of three variables, three canonical functions were extracted. Some of the computations utilized in this extraction are explained elsewhere by Thompson (1984, pp. 11-14) and are illustrated in the computer program, CANBAK (Thompson, 1982).

Table 2 depicts the computation of the synthetic variables scores actually correlated in CCA. The computations for Function I are presented here; readers may wish to themselves compute the synthetic scores for Functions II and III. The weights for the criterion variables on Function I were: (a) .6717, CHA6; (b) .3570,

INT6; (c) .4214, OTH6. Thus, the weighted and aggregated criterion Z-scores of subject 1 yield a synthetic criterion score for this subject of .81896 ((.6717 x .5654) + (.3570 x -.2868) + (.4214 x 1.2852) = .3798 - .1024 + .5416). The weights for the predictor variables on Function I were: (a) .4494, CHA2; (b) .7200, INT2; (c) .2228, OTH2. Thus, the weighted and aggregated predictor Z-scores of subject 1 yield a synthetic criterion score for this subject of .70814 ((.4494 x -.3317) + (.7200 x .9202) + (.2228 x .8738) = -.1491 + .6625 + .1947).

INSERT TABLE 2 ABOUT HERE.

The bivariate correlation between the synthetic scores on Function I is nothing more (or less) than the canonical correlation coefficient (Rc). Thus, for Function I, Rc = .932195 = $r_{CRIT1xPRED1}$. This is graphically illustrated in Figure 1. The synthetic variables are themselves Z-scores, the a intercept for the regression line is at the 0,0 coordinate, and the slope of the regression line is also Rc. Similarly, the bivariate correlation between the two sets of synthetic scores on Function II is the Rc for that function. The canonical function coefficients are specifically computed to optimize the calculated relationships between the synthetic variables on each function.

INSERT FIGURE 1 ABOUT HERE.

Table 3 presents most of the results for the full canonical analysis. The structure coefficients presented in the table have the same meaning in a canonical analysis as in other analyses, i.e., structure coefficients are always bivariate correlation coefficients between observed variable scores (e.g., "CHA6", "OTH6") and a synthetic variable (e.g., "CRIT1") created using weights. For example, if regression predictors are multiplied by regression weights and the products are summed for each individual, the correlation between scores on a given observed predictor and the synthetic variables scores ($\hat{Y}$) is the structure coefficient for that predictor. Similarly, in the canonical case a structure coefficient on a given function is the bivariate correlation between a given criterion or predictor variable and the synthetic variable involving the variable set to which the variable belongs. For example, since "ZCHA6" was a criterion variable in the Table 2 example, the correlation (+.8098) between "ZCHA6" and "CRIT1" is the structure coefficient for "ZCHA6" on Function I.

INSERT TABLE 3 ABOUT HERE.

In terms of actual contemporary analytic practice, Eason, Daniel and Thompson (1990) found that in about one-third of the published canonical studies researchers only report and interpret

function coefficients. But structure coefficients are vitally important in interpreting results in other analytic cases, such as factor analysis (Gorsuch, 1983, p. 207) and multiple regression analysis (Cooley & Lohnes, 1971, pp. 54-55; Thompson & Borrello, 1985). Similarly, with respect to CCA it is important not to interpret results based solely on function coefficients (Kerlinger & Pedhazur, 1973, p. 344; Levine, 1977, p. 20; Meredith, 1964, p. 55), though Harris (1989) may disagree. The structure and function coefficients for a variable set will be equal only if the variables in a set are all exactly uncorrelated with each other (Thompson, 1984, pp. 22-23), as would be the case, for example, if the variables in a set consisted of scores on orthogonally rotated principal components.

It would be dangerous to conclude that consulting either function or structure coefficients will always yield the same interpretations for a given data set. For example, Sexton, McLean, Boyd, Thompson and McCormick (1988) present a canonical analysis in which one variable had a function coefficient of +0.02 on Function I, but the same variable had a structure coefficient of +0.89 on the same function. It is important to know when either set of coefficients suggests that a variable may be noteworthy.

Canonical Correlation Analysis (CCA) as a General Method Subsuming Univariate and Multivariate Parametric Methods

Long ago Cohen (1968, p. 426) noted that ANOVA and ANCOVA are special cases of multiple regression analysis, and argued that in this realization "lie possibilities for more relevant and therefore more powerful exploitation of research data." However, Knapp (1978) offered mathematical proofs that CCA subsumes parametric methods, including both univariate and multivariate analyses. This realization is a basis for understanding how parametric methods are interrelated, which students often find to be helpful.

Three important insights can be gained from this perspective. All classical parametric methods (*t*-tests, ANOVA, MANOVA, etc.) are procedures that either implicitly or explicitly (a) use least squares weights, (b) focus on synthetic variables, and (c) yield effect sizes analogous to r^2. Put differently, all classical analytic methods are correlational. As Keppel and Zedeck (1989) repeatedly emphasize, the power to make causal inferences inures to design features and not the analytic method selected, since conventional parametric analyses are all correlational.

It is beyond the scope of the present treatment to explore all the possible relationships among analytic techniques. Knapp (1978) offers the mathematical proofs and additional concrete illustrations have been offered elsewhere (Thompson, 1988). However, a brief exploration of a couple of linkages may be useful to the reader. The Appendix A SAS program can be run using the Table 1 data to yield additional insights.

The linkage of CCA and multiple regression analysis is particularly easy to see, since both procedures are happily explicitly named correlational procedures. Suppose that the

researcher wanted to predict "INT6" with "CHA2", "INT2" and "OTH2", and did so using both regression and canonical correlation procedures. When the Appendix A SAS program file was applied to the Table 1 data to yield these analyses, PROC REG computed the squared multiple correlation coefficient to be .4016 (F = 1.789, df = 3/8, p = .2269); PROC CANCORR computed the squared canonical correlation coefficient to be .401566 (F = 1.7894, df = 3/8, p = .2269). These results differ only as to the arbitrary number of digits used to report the identical results.

The relationships between the beta weights produced by PROC REG and the function coefficients produced by PROC CANCORR are a bit harder to see. These results are presented in Table 4. The table also illustrates that weights are related, though they are standardized using a different metric. Thompson and Borrello (1985) provide more detail.

INSERT TABLE 4 ABOUT HERE.

The linkages between CCA and factorial ANOVA illustrate how CCA subsumes OVA methods (i.e., ANOVA, ANCOVA, MANOVA, MANCOVA) generally. For the 3 x 2 factorial ANOVA involving the IQ and experimental group assignment data presented in Table 1, PROC ANOVA yielded the following results for the three omnibus hypotheses: (a) IQ, F = 3.90; (b) experimental assignment, F = 1.85; (c) two-way interaction, F = 1.08. The Appendix A program was used to test four related canonical models, and the lambdas calculated from PROC CANCORR were then expressed as F's, using the process summarized in Table 5.

INSERT TABLE 5 ABOUT HERE.

These illustrative results correctly indicate that you can do regression with CCA, though you can't do CCA with regression. You can do factorial ANOVA with CCA, though you can't do CCA with ANOVA. The same relationship holds with other parametric methods (e.g., t-tests, ANCOVA, MANOVA). In short, CCA is a general parametric method subsuming other parametric methods as special cases.

Evaluating Result Generalizability

It is very important to use *use statistical formulas (e.g., Wherry, 1931) or (better yet) empirical methods to evaluate the generalizability of the results in hand*. The business of science is formulating generalizable insight. No one study, taken singly, establishes the basis for such insight. As Neale and Liebert (1986, p. 290) observe:

> No one study, however shrewdly designed and carefully executed, can provide convincing support for a causal hypothesis or theoretical statement... Too many possible (if not plausible) confounds, limitations on generality, and alternative

> interpretations can be offered for any one observation. Moreover, each of the basic methods of research (experimental, correlational, and case study) and techniques of comparison (within- or between-subjects) has intrinsic limitations. How, then, does social science theory advance through research? The answer is, by collecting a diverse body of evidence about any major theoretical proposition.

Evaluating the generalizability of canonical results to other samples of subjects or of variables is a difficult task, but a task which the serious scholar can ill-afford to shirk. It must be emphasized that statistical significance testing **does not inform the researcher regarding the likelihood that CCA Rc^2** (i.e., effect sizes) or other coefficients (e.g., function or structure coefficients) will be replicable in future research (Carver, 1978).

With respect to the replicability of CCA *effect sizes*, these estimates appear to be reasonably stable if the researcher uses at least 10 subjects per variable (Thompson, 1990a). Furthermore, several statistical corrections of the effect sizes can be invoked. One might employ Wherry's (1931) correction formula to Rc^2, as suggested by Cliff (1987, p. 446). But as incisively implied by Stevens (1986, pp. 78-84) with respect to the related regression case, the correction suggested by Herzberg (1969) may be especially useful, though it is more conservative. For example, for the Function I results reported in Table 3, the Wherry correction can be evaluated as:

$$
\begin{aligned}
&Rc^2 - ((1 - Rc^2) * (VTot / (NTot - VTot - 1)))\\
&.869 - ((1 - .869) * (6 / (12 - 6 - 1)))\\
&.869 - (.131 * (6 / 5))\\
&.869 - (.131 * 1.2)\\
&.869 - .1572 = .7118
\end{aligned}
$$

Efforts to estimate the sampling specificity of *coefficients for specific variables* are more difficult, or at least more tedious. CCA function and structure coefficients appear to be less stable than CCA omnibus effect sizes (Rc^2's), though both appear to be equally unstable (Thompson, 1989b). Thus, it is especially important to evaluate the generalizability of these coefficients.

Some researchers randomly split their sample data, conduct separate analyses for the two subgroups, and then <u>subjectively</u> compare the results to determine if they appear to be similar. Two points need to be emphasized about such an approach. Such procedures almost always overestimate the invariance or generalizability of results, as Thompson (1984, p. 46) explains. Also, it is emphasized that inferences regarding replicability must be made **empirically** rather than subjectively. Crowley and Thompson (1991) explain one strategy for making such comparisons. Functions that appear to be quite different may in fact yield quite similar synthetic variable scores--apparent differences in functions yielding comparable values for the synthetic variables actually related in canonical analysis are not very noteworthy (Thompson, 1989c). Cliff (1987, pp. 177-178) suggests that such cases involve

"insensitivity" of the weights to departures from least squares constraints.

Empirical methods for evaluating the generalizability of CCA coefficients are explored by Thompson (1984, pp. 41-47; 1990c). A sophisticated logic called the "bootstrap," popularized by Efron and more recently by Lunneborg (e.g., 1987) may be especially useful. The bottom line is that in all studies, CCA or not, results from a single study must be interpreted with some caution.

The data reported by Holzinger and Swineford (1939, pp. 81-91), used with some frequency to illustrate multivariate statistical analyses (e.g., Gorsuch, 1983, passim; Jöreskog & Sörbom, 1986, pp. III.106-III.122), are used here to illustrate one strategy for evaluating generalizability of multivariate results. Crowley and Thompson (1991) illustrate some alternatives. These data were selected for use in the examples because they are widely available, and interested readers can therefore readily replicate the analyses described here. Table 6 presents results from a canonical analysis relating the five verbal test scores from the Holzinger-Swineford (1939) data to scores on the four visual perception tests.

INSERT TABLE 6 ABOUT HERE.

The strategy illustrated here invokes a *modelling* procedure. First, a population of 10,000 cases were generated subject to the restriction that means, SDs, and correlation coefficients be the same as those in the Holzinger-Swineford (1939) data. This was done using the program written by Morris (1975).

Next, random samples of size n=301 were drawn from the population to explore the effects of sampling error. For illustrative purpose, five samples were drawn in the present paper. These results are presented in Table 7.

INSERT TABLE 7 ABOUT HERE.

In order to compare apples with apples, the functions must be examined to determine (a) whether the functions need to be "reflected" so that they are oriented in the same direction, and (b) whether the functions change orders across samples. In the present example, function and structure coefficients for sample #1, Function I, and for samples #1, #3, #4, and #5, Function II, were reflected by multiplication by negative one. Such "reflection" is always available to the researcher, whenever it facilitates interpretation, and is perfectly legitimate (Gorsuch, 1983).

Examination of the results suggested that at least the first two functions appeared in the same order across samples. This is important to check when canonical correlation coefficients are homogeneous, since function order may arbitrarily fluctuate in such cases.

Finally, descriptive statistics for parameter estimates are

computed. These are presented for the present example in Table 8. The SDs from such analyses are analogous to standard errors, and are of particular interest. The present example makes clear that (a) the Rc^2 effect size is inflated somewhat by sampling error (e.g., 19.2% versus the mean of 25.1%); (b) estimates of coefficients at the variable level are more subject to sampling error; and (c) results are more stable for functions with larger effect sizes. These results are consistent with previous research (cf. Thompson, 1990a, 1990c).

INSERT TABLE 8 ABOUT HERE.

Summary

As Stevens (1986, p. 373, emphasis omitted) notes, CCA "is appropriate if the wish is to parsimoniously describe the number and nature of mutually independent relationships between the two [variable] sets... Since the [function] combinations are uncorrelated, we will obtain a very nice additive partitioning of the total between association." The present article has explained the basic logic of canonical correlation analysis. It was noted that all parametric analytic methods are correlational, and that all parametric tests can be conducted using canonical analysis, since canonical analysis subsumes parametric methods as special cases. Canonical analysis is potent because it does not require the researcher to discard variance of any of the variables, and because the analysis honors the complexity of a reality in which variables interact simultaneously.

References

Benton, R. (1991). Statistical power considerations in ANOVA. In B. Thompson (Ed.), Advances in educational research: Substantive findings, methodological developments (Vol. 1, pp. 119-132). Greenwich, CT: JAI Press.

Carver, R. P. (1978). The case against statistical significance testing. Harvard Educational Review, 48(3), 378-399.

Cliff, N. (1987). Analyzing multivariate data. San Diego: Harcourt Brace Jovanovich.

Cohen, J. (1968). Multiple regression as a general data-analytic system. Psychological Bulletin, 70, 426-443.

Cooley, W. W., & Lohnes, P. R. (1971). Multivariate data analysis. New York: John Wiley and Sons.

Cramer, E. M., & Nicewander, W. A. (1979). Some symmetric, invariant measures of multivariate association. Psychometrika, 44, 43-54.

Crowley, S.L., & Thompson, B. (1991, April). Evaluating the replicability of multivariate assessment and evaluation results: A review of various strategies. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Eason, S. (1991). Why generalizability theory yields better results than classical test theory: A primer with concrete examples. In B. Thompson (Ed.), Advances in educational research: Substantive findings, methodological developments (Vol. 1, pp. 83-98). Greenwich, CT: JAI Press.

Eason, S., Daniel, L., & Thompson, B. (1990, January). A review of practice in a decade's worth of canonical correlation analysis studies. Paper presented at the annual meeting of the Southwest Educational Research Association, Austin, TX.

Elmore, P.B., & Woehlke, P.L. (1988). Statistical methods employed in American Educational Research Journal, Educational Researcher, and Review of Educational Research from 1978 to 1987. Educational Researcher, 17(9), 19-20.

Fish, L.J. (1988). Why multivariate methods are usually vital. Measurement and Evaluation in Counseling and Development, 21, 130-137.

Gorsuch, R. L. (1983). Factor analysis (2nd ed.). Hillsdale, NJ: Erlbaum.

Harris, R.J. (1989). A canonical cautionary. Multivariate Behavioral Research, 24, 17-39.

Herzberg, P.A. (1969). The parameters of cross validation. Psychometrika, Monograph supplement, No. 16.

Hinkle, D.E., Wiersma, W., & Jurs, S.G. (1979). Applied statistics for the behavioral sciences. Chicago: Rand McNally.

Jöreskog, K.G., & Sörbom, D. (1986). LISREL 6: Analysis of linear structural relationships by maximum likelihood, instrumental variables, and least squares methods (4th ed.). Mooresville, IN: Scientific Software.

Keppel, G., & Zedeck, S. (1989). Data analysis for research designs. New York: W. H. Freeman.

Kerlinger, F. N., & Pedhazur, E. J. (1973). Multiple regression in

behavioral research. New York: Holt, Rinehart and Winston.

Knapp, T. R. (1978). Canonical correlation analysis: A general parametric significance testing system. Psychological Bulletin, 85, 410-416.

Levine, M. S. (1977). Canonical analysis and factor comparison. Newbury Park: SAGE.

Lunneborg, C.E. (1987). Bootstrap applications for the behavioral sciences. Seattle: University of Washington.

Maxwell, S. (in press). Recent developments in MANOVA applications. In B. Thompson (Ed.), Advances in social science methodology (Vol. 2). Greenwich, CT: JAI Press.

Meredith, W. (1964). Canonical correlations with fallible data. Psychometrika, 29, 55-65.

Morenda, P., Novack, H., & Bonaventure, E. (1976). Multivariate analysis of the California test of mental maturity, primary forms. Psychological Reports, 38, 487-493.

Morris, J.D. (1975). A computer program to create a population with any desired centroid and covariance matrix. Educational and Psychological Measurement, 35, 707-710.

Neale, J.M., & Liebert, R.M. (1986). Science and behavior: An introduction to methods of research (3rd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Nunnally, J.C. (1978). Psychometric theory (2nd ed.). New York: McGraw-Hill.

Sexton, J.D., McLean, M., Boyd, R.D., Thompson, B., & McCormick, K. (1988). Criterion-related validity of a new standardized developmental measure for use with infants who are handicapped. Measurement and Evaluation in Counseling and Development, 21, 16-24.

Stevens, J. (1986). Applied multivariate statistics for the social sciences. Hillsdale, NJ: Erlbaum.

Tatsuoka, M. M. (1973). Multivariate analysis in educational research. In F. N. Kerlinger (Ed.), Review of research in education (pp. 273-319). Itasca, IL: Peacock.

Thompson, B. (1980, April). Canonical correlation: Recent extensions for modelling educational processes. Paper presented at the annual meeting of the American Educational Research Association, Boston. (ERIC Document Reproduction Service No. ED 199 269)

Thompson, B. (1982). CANBAK: A computer program which performs stepwise canonical correlation analysis. Educational and Psychological Measurement, 42, 849-851.

Thompson, B. (1984). Canonical correlation analysis: Uses and interpretation. Newbury Park: SAGE.

Thompson, B. (1986, November). Two reasons why multivariate methods are usually vital. Paper presented at the annual meeting of the Mid-South Educational Research Association, Memphis.

Thompson, B. (1987, April). The use (and misuse) of statistical significance testing: Some recommendations for improved editorial policy and practice. Paper presented at the annual meeting of the American Educational Research Association, Washington, D.C. (ERIC Document Reproduction Service No. ED 287

868)
Thompson, B. (1988, April). Canonical correlation analysis: An explanation with comments on correct practice. Paper presented at the annual meeting of the American Educational Research Association, New Orleans. (ERIC Document Reproduction Service No. ED 295 957)
Thompson, B. (1989a, August). Applications of multivariate statistics: A bibliography of canonical correlation analysis studies. Paper presented that the annual meeting of the American Psychological Association, New Orleans. (ERIC Reproduction Service No. ED 311 070)
Thompson, B. (1989b, March). Invariance of multivariate results: A Monte Carlo study of canonical coefficients. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service No. ED 306 226)
Thompson, B. (1989c). Statistical significance, result importance, and result generalizability: Three noteworthy but somewhat different issues. Measurement and Evaluation in Counseling and Development, 22, 2-6.
Thompson, B. (1990a). Finding a correction for the sampling error in multivariate measures of relationship: A Monte Carlo study. Educational and Psychological Measurement, 50, 15-31.
Thompson, B. (1990b). MULTINOR: A FORTRAN program that assists in evaluating multivariate normality. Educational and Psychological Measurement, 50, 845-848.
Thompson, B. (1990c, April). Variable importance in multiple and canonical correlation/regression. Paper presented at the annual meeting of the American Educational Research Association, Boston. (ERIC Document Reproduction Service No. ED 317 615)
Thompson, B. (1991). Review of Data analysis for research designs by G. Keppel & S. Zedeck. Educational and Psychological Measurement, 51.
Thompson, B., & Borrello, G. M. (1985). The importance of structure coefficients in regression research. Educational and Psychological Measurement, 45, 203-209.
Thompson, B., & Daniel, L. (1991, April). Canonical correlation analyses that incorporate measurement and sampling error considerations. Paper presented at the annual meeting of the American Educational Research Association, Chicago.
Thompson, B., Webber, L., & Berenson, G.S. (1988). Validity of a children's health locus of control measure: A "Heart Smart" study. American Journal of Health Promotion, 3(2), 44-49.
Thorndike, R. M. (1976, April). Strategies for rotating canonical components. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service No. ED 123 259)
Wherry, R.J. (1931). A new formula for predicting the shrinkage of the coefficient of multiple correlation. Annals of Mathematical Statistics, 2, 440-451.
Wood, D.A., & Erskine, J.A. (1976). Strategies in canonical correlation with application to behavioral data. Educational

and Psychological Measurement, _36_, 861-878.

Table 1
Random Sample (n=12) of Health Locus of Control Data
with Hypothetical IQ and Experimental Group Assignments

ID	CHA6	INT6	OTH6	CHA2	INT2	OTH2	IQ	IQGRP	EXPERGRP	CIQGRP1	CIQGRP2	CEXGRP1	CIQBYEX1	CIQBYEX2
1	20	17	19	7	7	7	68	1	1	-1	-1	1	-1	-1
2	21	20	15	8	7	5	89	1	1	-1	-1	1	-1	-1
3	17	15	20	7	6	7	50	1	2	-1	-1	-1	1	1
4	16	14	13	8	5	5	85	1	2	-1	-1	-1	1	1
5	20	20	15	8	7	5	90	2	1	0	2	1	0	2
6	14	21	15	7	5	7	109	2	1	0	2	1	0	2
7	14	19	14	7	5	6	102	2	2	0	2	-1	0	-2
8	14	23	10	6	6	6	108	2	2	0	2	-1	0	-2
9	21	14	12	8	5	2	111	3	1	1	-1	1	1	-1
10	19	12	10	7	6	4	140	3	1	1	-1	1	1	-1
11	24	24	19	8	8	8	120	3	2	1	-1	-1	-1	1
12	18	18	9	6	6	5	143	3	2	1	-1	-1	-1	1

Table 2
Variables in Z-score Form and Synthetic Composite Scores on Function I

OBS	ZCHA6	ZINT6	ZOTH6	ZCHA2	ZINT2	ZOTH2	CRIT1	PRED1
1	.5654	-.2868	1.2852	-.3317	.9202	.8738	.81896	.70814
2	.8738	.5075	.2029	.9950	.9202	-.3598	.85358	1.02950
3	-.3598	-.8164	1.5558	-.3317	-.0837	.8738	.12251	-.01460
4	-.6682	-1.0811	-.3382	.9950	-1.0875	-.3598	-.97730	-.41598
5	.5654	.5075	.2029	.9950	.9202	-.3598	.64644	1.02950
6	-1.2849	.7722	.2029	-.3317	-1.0875	.8738	-.50189	-.73735
7	-1.2849	.2427	-.0676	-.3317	-1.0875	.2570	-.80495	-.8747[illegible]
8	-1.2849	1.3018	-1.1499	-1.6583	-.0837	.2570	-.88295	-.74822
9	.8738	-1.0811	-.6088	.9950	-1.0875	-2.2101	-.05561	-.82823
10	.2570	-1.6107	-1.1499	-.3317	-.0837	-.9766	-.88697	-.42585
11	1.7989	1.5665	1.2852	.9950	1.9240	1.4905	2.30918	2.16449
12	-.0514	-.0221	-1.4205	-1.6583	-.0837	-.3598	-.64101	-.885[illegible]

Table 3
Canonical Solution for the Table 1 Data

Variable/ Coef.	Function I Func.	Str.	Str.^{2}	Function II Func.	Str.	Str.^{2}	Function III Func.	Str.	Str.^{2}	h^2
CHA6	.6717	.8098	65.6%	-.8174	-.5633	31.7%	-.0266	.1643	2.7%	100.0%[a]
INT6	.3570	.4428	19.6%	.2433	.3901	15.2%	-.9253	-.8073	65.2%	100.0%
OTH6	.4214	.7071	50.0%	.7837	.5673	32.2%	.6099	.4220	17.8%	100.0%
Adequacy			45.1%[b]			26.4%			28.6%	
Rd			39.2%[c]			21.8%			4.6%	
Rc^2			86.9%			82.5%			16.2%	
Rd			38.3%			20.7%			5.0%	
Adequacy			44.0%			25.2%			30.8%	
CHA2	.4494	.5564	31.0%	.1669	-.2017	4.1%	.9721	.8061	65.0%	100.0%
INT2	.7200	.9082	82.5%	-.6422	-.1324	1.8%	-.6576	-.3971	15.8%	100.0%
OTH2	.2228	.4314	18.6%	1.1368	.8345	69.6%	.1307	-.3427	11.7%	100.0%

[a]Canonical communality (h^2) coefficients are directly analogous to the factor analytic coefficients of the same name, and indicate how much of the variance of an observed variable is contained within the set of synthetic variables. For example, the communality coefficient for "CHA6" equals 65.6% + 31.7% + 2.7%.

[b]An adequacy coefficient indicates how adequately the synthetic scores on a function do at reproducing the variance in a set of variables, and equals the mean of the squared structure coefficients on the variable. Thus, the adequacy coefficient for the criterion variable set on Function I equals (65.6% + 19.6% + 50.0%) / 3 = 135.2 / 3 = 45.1%.

[c]A redundancy (Rd) coefficient equals an adequacy coefficient times Rc^2, e.g., 45.1% times 86.9% equals 39.2%.

Table 4
The Relationship Between
Regression beta Weights and CCA Function Coefficients

Variable	beta		Function Coefficients
CHA2	-0.07129038	/ R =	-0.1125
INT2	0.28335280	/ R =	0.4471
OTH2	0.45229076	/ R =	0.7137

Note. R = Rc = 0.633692. The weights are reported to the same number of decimal places produced on the SAS output.

Table 5
Three Steps to Convert CCA Results to Factorial ANOVA F's

Step #1: Get CCA lambda for 4 sets of orthogonal contrast variables.

Model	Predictors					lambda
1	CIQGRP1	CIQGRP2	CEXGRP1	CIQBYEX1	CIQBYEX2	.33717579
2			CEXGRP1	CIQBYEX1	CIQBYEX2	.77521614
3	CIQGRP1	CIQGRP2		CIQBYEX1	CIQBYEX2	.44092219
4	CIQGRP1	CIQGRP2	CEXGRP1			.45821326

Step #2: Convert lambdas to ratios for each effect.

Effect	Ratio	Full Model lambda		lambda w/o Effect		Ratio
IQ	1 / 2	.33717579	/	.77521614	=	.434944
Exp. Assignment	1 / 3	.33717579	/	.44092219	=	.764705
IQ x Exp. Interaction	1 / 4	.33717579	/	.45821326	=	.735849

Step #3: Convert ratios to ANOVA F's, by the algorithm, F = ((1 - effect ratio) / ratio) x (df error / df effect)

```
IQ                      ((1 - .434944) / .434944) x (6 / 2)
                        (     .565055  / .434944) x  3
                                     1.299145     x  3
                                                    3.897435

Exp. Assignment         ((1 - .764705) / .764705) x (6 / 1)
                        (     .235294  / .764705) x  6
                                      .307692     x  6
                                                    1.846153

IQ x Exp. Interaction   ((1 - .735849) / .735849) x (6 / 2)
                        (     .264150  / .735849) x  3
                                      .358974     x  3
                                                    1.076923
```

Table 6
Canonical Results for the Holzinger and Swineford (1939) Data
(N=301)

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	0.06	-0.72	51.9%	0.81	0.49	24.3%	1.01	0.45	20.1%	-1.00	-0.19	3.7%
T6	-0.51	-0.86	74.7%	0.05	0.32	10.3%	-1.23	-0.35	12.0%	-0.85	-0.16	2.6%
T7	0.34	-0.69	47.0%	1.16	0.59	35.2%	-0.23	0.00	0.0%	1.17	0.38	14.2%
T8	-0.47	-0.80	64.5%	-0.58	0.11	1.1%	0.16	0.18	3.1%	0.59	0.37	13.6%
T9	-0.53	-0.87	75.9%	-1.09	0.04	0.1%	0.42	0.23	5.1%	0.20	0.05	0.3%
Adequacy			62.8%			14.2%			8.1%			6.9%
Rd			12.1%			0.4%			0.1%			0.0%
Rc^2			19.2%			2.7%			0.7%			0.2%
Rd			8.5%			0.5%			0.1%			0.0%
Adequacy			44.1%			18.7%			18.4%			18.8%
T1	-0.78	-0.95	89.8%	0.51	0.08	0.7%	-0.70	-0.31	9.4%	0.08	0.01	0.0%
T2	-0.17	-0.48	22.8%	0.03	-0.16	2.7%	0.56	0.49	24.2%	0.91	0.71	50.3%
T3	-0.25	-0.60	35.4%	0.18	0.03	0.1%	0.80	0.63	39.8%	-0.68	-0.50	24.7%
T4	-0.05	-0.53	28.3%	-1.13	-0.84	71.3%	-0.16	-0.03	0.1%	-0.19	-0.06	0.3%

Table 7
Canonical Results for Five Samples (n=301) from a Population of 10,000 Modelled on the Holzinger and Swineford (1939) Data

N=301 SAMPLE #1

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	-0.490	-0.853	72.7%	1.211	0.467	21.8%	-0.054	0.230	5.3%	0.017	0.032	0.1%
T6	-0.557	-0.867	75.1%	-0.232	0.051	0.3%	-1.494	-0.212	4.5%	-0.473	-0.181	3.3%
T7	0.594	-0.647	41.8%	0.844	0.352	12.4%	0.644	0.308	9.5%	-0.014	0.010	0.0%
T8	-0.323	-0.782	61.2%	-0.708	-0.050	0.3%	0.056	0.187	3.5%	1.319	0.528	27.9%
T9	-0.284	-0.816	66.6%	-1.008	-0.113	1.3%	1.078	0.451	20.4%	-0.770	-0.282	8.0%
Adequacy			63.5%			7.2%			8.6%			7.9%
Rd			18.8%			0.5%			0.4%			0.0%
Rc^2			29.6%			6.7%			4.3%			0.4%
Rd			12.7%			1.5%			0.8%			0.1%
Adequacy			42.8%			21.7%			17.8%			17.7%
T1	-0.804	-0.929	86.3%	0.091	-0.195	3.8%	-0.829	-0.211	4.5%	-0.410	-0.232	5.4%
T2	-0.147	-0.481	23.2%	-0.431	-0.484	23.4%	-0.047	0.054	0.3%	0.992	0.729	53.1%
T3	-0.363	-0.686	47.0%	0.661	0.278	7.7%	0.835	0.671	45.0%	0.137	0.051	0.3%
T4	0.176	-0.381	14.5%	-0.867	-0.721	52.0%	0.578	0.462	21.4%	-0.502	-0.348	12.1%

N=301 SAMPLE #2

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	0.016	-0.754	56.8%	-0.140	0.169	2.9%	0.932	0.316	10.0%	-0.097	-0.115	1.3%
T6	-0.378	-0.893	79.8%	-1.073	-0.170	2.9%	0.930	0.328	10.8%	0.015	-0.091	0.8%
T7	-0.044	-0.826	68.2%	1.665	0.519	26.9%	0.202	0.202	4.1%	-0.073	-0.043	0.2%
T8	-0.344	-0.826	68.3%	-0.196	0.081	0.7%	-0.574	-0.134	1.8%	1.209	0.521	27.1%
T9	-0.395	-0.897	80.5%	-0.166	0.042	0.2%	-1.366	-0.207	4.3%	-0.979	-0.365	13.4%
Adequacy			70.7%			6.7%			6.2%			8.6%
Rd			16.4%			0.3%			0.1%			0.0%
Rc^2			23.2%			4.6%			1.2%			0.5%
Rd			8.3%			0.9%			0.2%			0.1%
Adequacy			35.8%			20.6%			20.5%			23.1%
T1	-0.918	-0.965	93.2%	0.044	-0.146	2.1%	0.602	0.214	4.6%	-0.279	0.030	0.1%
T2	0.012	-0.337	11.3%	0.246	0.094	0.9%	0.073	-0.088	0.8%	1.058	0.933	87.0%
T3	-0.277	-0.541	29.3%	0.362	0.158	2.5%	-0.968	-0.825	68.1%	-0.228	0.026	0.1%
T4	0.104	-0.305	9.3%	-1.056	-0.877	76.9%	-0.267	-0.295	8.7%	0.120	0.228	5.2%

N=301 SAMPLE #3

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	-0.312	-0.832	69.2%	-[illegible].454	0.062	0.4%	1.150	0.303	9.2%	0.852	0.444	19.7%
T6	-0.106	-0.713	50.9%	-0.298	0.204	4.2%	-0.783	-0.403	16.2%	0.010	0.187	3.5%
T7	0.308	-0.664	44.1%	1.638	0.720	51.9%	0.186	-0.051	0.3%	0.271	0.177	3.1%
T8	-0.594	-0.865	74.9%	-0.220	0.223	5.0%	0.237	0.144	2.1%	-1.213	-0.420	17.6%
T9	-0.430	-0.827	68.3%	-0.371	0.113	1.3%	-0.880	-0.355	12.6%	0.186	0.339	11.5%
Adequacy			61.5%			12.5%			8.1%			11.1%
Rd			16.5%			0.7%			0.1%			0.1%
Rc^2			26.8%			5.4%			1.4%			0.8%
Rd			11.6%			1.0%			0.3%			0.2%
Adequacy			43.5%			19.1%			19.4%			18.1%
T1	-0.870	-0.972	94.5%	0.258	-0.06[illegible]	0.4%	-0.723	-0.225	5.1%	0.281	0.027	0.1%
T2	-0.237	-0.578	33.4%	0.710	0.299	8.9%	0.789	0.649	42.1%	-0.352	-0.394	15.5%
T3	-0.089	-0.480	23.0%	-0.571	-0.555	30.8%	0.628	0.539	29.1%	0.730	0.413	17.1%
T4	0.053	-0.478	22.9%	-0.809	-0.602	36.2%	-0.113	0.117	1.4%	-0.877	-0.629	39.6%

N=301 SAMPLE #4

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	0.366	-0.531	28.2%	0.859	0.719	51.7%	1.020	0.415	17.2%	0.515	-0.015	0.0%
T6	-0.523	-0.733	53.7%	0.100	0.552	30.5%	-1.137	-0.366	13.4%	0.641	0.041	0.2%
T7	0.741	-0.427	18.2%	0.963	0.796	63.4%	-0.450	-0.101	1.0%	-0.950	-0.314	9.9%
T8	-0.454	-0.661	43.7%	-0.216	0.473	22.4%	0.250	0.162	2.6%	0.907	0.274	7.5%
T9	-0.936	-0.831	69.0%	-0.961	0.351	12.3%	0.384	0.196	3.9%	-1.128	-0.385	14.8%
Adequacy			42.6%			36.0%			7.6%			6.5%
Rd			11.4%			2.6%			0.1%			0.1%
Rc^2			26.8%			7.2%			1.9%			0.9%
Rd			13.2%			1.0%			0.4%			0.2%
Adequacy			49.2%			14.[illegible]%			20.4%			16.0%
T1	-0.474	-0.816	66.6%	1.061	0.576	33.2%	-0.084	0.006	0.0%	-0.047	-0.049	0.2%
T2	0.027	-0.372	13.8%	-0.019	-0.121	1.5%	0.945	0.811	65.8%	-0.554	-0.434	18.9%
T3	-0.304	-0.685	46.[illegible]%	-0.348	-0.252	6.4%	0.343	0.356	12.7%	0.998	0.583	34.0%
T4	-0.497	-0.834	69.6%	-0.744	-0.402	16.1%	-0.621	-0.179	3.2%	-0.526	-0.332	11.0%

N=301 SAMPLE #5

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	-0.117	-0.759	57.6%	0.019	-0.042	0.2%	0.955	0.574	33.0%	-0.710	-0.113	1.3%
T6	-0.515	-0.874	76.4%	-0.434	-0.232	5.4%	-0.971	-0.139	1.9%	-0.992	-0.345	11.9%
T7	0.124	-0.761	57.9%	0.783	0.168	2.8%	0.777	0.361	13.0%	-0.376	-0.099	1.0%
T8	-0.492	-0.842	70.9%	0.696	0.454	20.6%	-0.[illegible]24	-0.020	0.0%	0.806	0.272	7.4%
T9	-0.181	-0.777	60.3%	-1.052	-0.431	18.6%	0.076	0.323	10.4%	1.306	0.247	6.1%
Adequacy			64.6%			9.5%			11.7%			5.5%
Rd			12.3%			0.2%			0.1%			0.0%
Rc^2			19.0%			2.0%			0.6%			0.2%
Rd			7.5%			0.5%			0.1%			0.0%
Adequacy			39.5%			22.8%			19.9%			17.8%
T1	-0.883	-0.961	92.3%	-0.412	-0.173	3.0%	0.513	0.004	0.0%	-0.360	-0.216	4.7%
T2	-0.119	-0.366	13.4%	1.007	0.907	82.3%	0.280	-0.043	0.2%	-0.273	-0.202	4.1%
T3	-0.260	-0.560	31.3%	0.123	0.212	4.5%	-0.123	-0.263	6.9%	1.052	0.757	57.2%
T4	0.082	-0.459	21.0%	-0.091	0.121	1.5%	-1.149	-0.851	72.4%	-0.313	-0.227	5.1%

Table 8
Descriptive Statistics Across the Five Samples

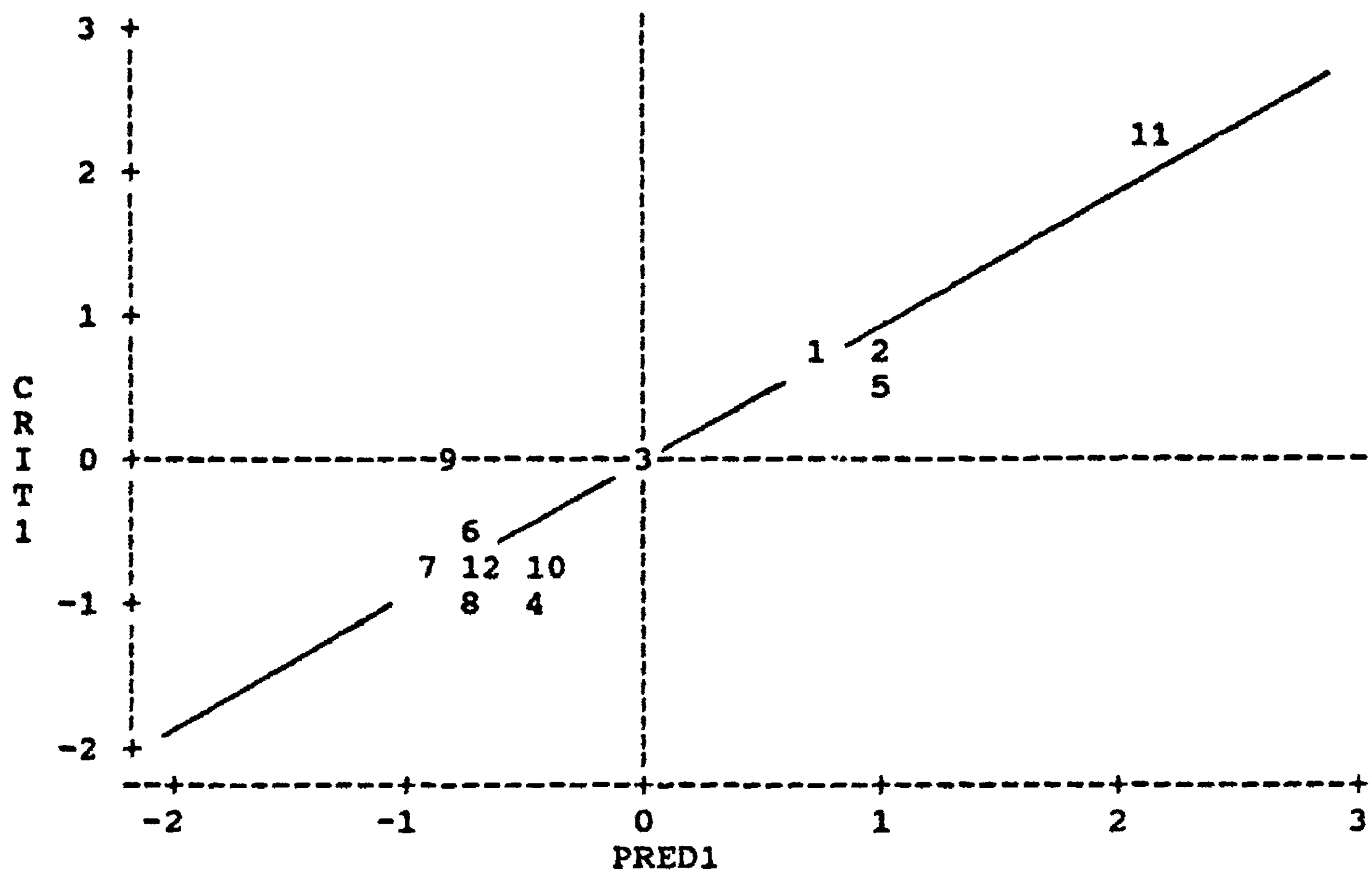
***MEANS

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	-0.107	-0.746	0.569	0.299	0.275	0.154	0.801	0.368	0.149	0.115	0.047	0.045
T6	-0.416	-0.816	0.672	-0.388	0.081	0.086	-0.691	-0.158	0.094	-0.160	-0.078	0.039
T7	0.345	-0.665	0.461	1.178	0.511	0.315	0.272	0.144	0.056	-0.229	-0.054	0.028
T8	-0.442	-0.795	0.638	-0.129	0.236	0.098	-0.131	0.068	0.020	0.606	0.235	0.175
T9	-0.457	-0.829	0.689	-0.712	-0.008	0.067	-0.142	0.082	0.103	-0.277	-0.090	0.107
Adequacy			0.606			0.144			0.084			0.079
Rd			0.151			0.009			0.002			0.000
Rc^2			0.251			0.052			0.019			0.006
Rd			0.107			0.010			0.004			0.001
Adequacy			0.422			0.197			0.196			0.185
T1	-0.790	-0.929	0.866	0.208	0.000	0.085	-0.104	-0.042	0.028	-0.163	-0.088	0.021
T2	-0.093	-0.427	0.190	0.302	0.139	0.234	0.408	0.277	0.218	0.174	0.126	0.357
T3	-0.258	-0.590	0.355	0.045	-0.032	0.104	0.143	0.096	0.324	0.538	0.366	0.217
T4	-0.016	-0.491	0.275	-0.713	-0.496	0.365	-0.314	-0.149	0.214	-0.420	-0.262	0.146

***SD

	I			II			III			IV		
	F	S	S^2	F	S	S^2	F	S	S^2	F	S	S^2
T5	0.292	0.114	0.156	0.630	0.280	0.199	0.434	0.119	0.098	0.536	0.207	0.076
T6	0.166	0.077	0.123	0.385	0.282	0.110	0.843	0.262	0.054	0.546	0.182	0.042
T7	0.290	0.136	0.169	0.391	0.232	0.230	0.431	0.187	0.049	0.415	0.160	0.037
T8	0.100	0.072	0.110	0.455	0.205	0.097	0.388	0.124	0.011	0.928	0.346	0.090
T9	0.283	0.039	0.065	0.368	0.259	0.074	0.878	0.310	0.061	0.914	0.315	0.033
Adequacy			0.095			0.110			0.018			0.019
Rd			0.028			0.009			0.001			0.000
Rc^2			0.037			0.019			0.013			0.003
Rd			0.023			0.003			0.002			0.000
Adequacy			0.045			0.030			0.010			0.024
T1	0.162	0.058	0.104	0.481	0.292	0.124	0.598	0.163	0.023	0.254	0.115	0.024
T2	0.100	0.090	0.083	0.511	0.463	0.306	0.392	0.377	0.273	0.701	0.584	0.304
T3	0.092	0.082	0.097	0.452	0.321	0.104	0.642	0.561	0.223	0.502	0.289	0.217
T4	0.244	0.182	0.216	0.328	0.345	0.265	0.571	0.438	0.264	0.326	0.279	0.128

Figure 1
Plot of CRIT1 by PRED1



Appendix A:
SAS Program for the Table 1 Data

```
DATA HLOCMECD; INFILE ABC;
INPUT ID 3-4 CHA6 6-7 INT6 9-10 OTH6 12-13 CHA2 15-16 INT2 18-19
  OTH2 21-22 IQ 24-26 IQGRP 28 EXPERGRP 30 CIQGRP1 32-33 CIQGRP2 35-36
  CEXGRP1 38-39 CIQBYEX1 41-42 CIQBYEX2 44-45;
PROC PRINT; VAR ID CHA6 INT6 OTH6 CHA2 INT2 OTH2 IQ IQGRP EXPERGRP
  CIQGRP1 CIQGRP2 CEXGRP1 CIQBYEX1 CIQBYEX2; RUN;
TITLE '1.  DESCRIPTION OF RAW DATA';
PROC CORR; VAR CHA6 INT6 OTH6 CHA2 INT2 OTH2 IQ IQGRP EXPERGRP
  CIQGRP1 CIQGRP2 CEXGRP1 CIQBYEX1 CIQBYEX2; RUN;
TITLE '2.  THE LOGIC OF CCA';
PROC CANCORR ALL; VAR CHA6 INT6 OTH6; WITH CHA2 INT2 OTH2;
data hlocnew; set hlocmecd;
zcha6=(cha6-18.166666667)/3.24270744;
zint6=(int6-18.083333333)/3.77692355;
zoth6=(oth6-14.250000000)/3.69582074;
zcha2=(cha2-07.250000000)/0.75377836;
zint2=(int2-06.083333333)/0.99620492;
zoth2=(oth2-05.583333333)/1.62135372;
crit1=(0.6717*zcha6)+(0.3570*zint6)+(0.4214*zoth6);
pred1=(0.4494*zcha2)+(0.7200*zint2)+(0.2228*zoth2);
crit2=(-.8174*zcha6)+(0.2433*zint6)+(0.7837*zoth6);
pred2=(0.1669*zcha2)+(-.6422*zint2)+(1.1368*zoth2);
crit3=(-.0266*zcha6)+(-.9253*zint6)+(0.6099*zoth6);
pred3=(0.9721*zcha2)+(-.6576*zint2)+(0.1307*zoth2);
proc print; var zcha6 zint6 zoth6 zcha2 zint2 zoth2
  crit1 pred1 crit2 pred2 crit3 pred3; run;
title '2a  AN r MATRIX WITH MANY REVELATIONS';
proc corr; var zcha6 zint6 zoth6 zcha2 zint2 zoth2
  crit1 pred1 crit2 pred2 crit3 pred3; run;
title '2b  THE 1ST FUNCTION IN GRAPHIC FORM';
proc plot; plot crit1*pred1=id/ vaxis=-3 to 7 by 1 vref=0
  haxis=-3 to 8 by 1 href=0; run;
TITLE '3.  CCA SUBSUMES PEARSON CORRELATION';
PROC CORR; VAR OTH6 OTH2;
PROC CANCORR ALL; VAR OTH6; WITH OTH2; RUN;
TITLE '4.  CCA SUBSUMES T-TESTS & ONE-WAY ANOVA';
PROC TTEST; CLASS EXPERGRP; VAR OTH6;
PROC ANOVA; CLASS EXPERGRP; MODEL OTH6=EXPERGRP;
PROC CANCORR ALL; VAR OTH6; WITH CEXGRP1; RUN;
TITLE '5.  CCA SUBSUMES FACTORIAL ANOVA';
PROC ANOVA; CLASS IQGRP EXPERGRP;
MODEL CHA6=IQGRP EXPERGRP IQGRP*EXPERGRP;
PROC CANCORR; VAR CHA6; WITH CIQGRP1 CIQGRP2 CEXGRP1 CIQBYEX1 CIQBYEX2;
PROC CANCORR; VAR CHA6; WITH                 CEXGRP1 CIQBYEX1 CIQBYEX2;
PROC CANCORR; VAR CHA6; WITH CIQGRP1 CIQGRP2         CIQBYEX1 CIQBYEX2;
PROC CANCORR; VAR CHA6; WITH CIQGRP1 CIQGRP2 CEXGRP1; RUN;
TITLE '6.  CCA SUBSUMES MULTIPLE REGRESSION';
PROC REG; MODEL INT6=CHA2 INT2 OTH2/ STB;
PROC CANCORR ALL; VAR INT6; WITH CHA2 INT2 OTH2; RUN;
```

```
TITLE '7.  CCA SUBSUMES FACTORIAL MANOVA';
PROC ANOVA; CLASS IQGRP EXPERGRP;
  MODEL CHA6 INT6=IQGRP EXPERGRP IQGRP*EXPERGRP; MANOVA H=_ALL_/SUMMARY;
PROC CANCORR ALL; VAR CHA6 INT6;
  WITH CIQGRP1 CIQGRP2 CEXGRP1 CIQBYEX1 CIQBYEX2;
PROC CANCORR ALL; VAR CHA6 INT6;
  WITH                CEXGRP1 CIQBYEX1 CIQBYEX2;
PROC CANCORR ALL; VAR CHA6 INT6;
  WITH CIQGRP1 CIQGRP2         CIQBYEX1 CIQBYEX2;
PROC CANCORR ALL; VAR CHA6 INT6;
  WITH CIQGRP1 CIQGRP2 CEXGRP1; RUN;
TITLE '8.  CCA SUBSUMES DISCRIMINANT';
PROC CANDISC ALL; VAR CHA6 INT6; CLASS EXPERGRP;
PROC CANCORR ALL; VAR CHA6 INT6; WITH CEXGRP1;
```

Note. The bulk of the program was executed as the first of two runs. The lower case commands required the results from the first run, since the relevant coefficients were not yet known. These commands were then added into the program, and the program was executed a second time. Of course, since the required values used on the lower case commands are presented here, for this particular example the reader can execute the full program in a single step.